DIFFERENT TYPES OF MATRICES IN INTUITIONISTIC FUZZY SOFT SET THEORY AND THEIR APPLICATION IN PREDICTING TERRORIST ATTACK

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Abstract:

The purpose of this paper is to introduce different types of intuitionistic fuzzy soft matrices along with some new operations in the parlance of intuitionistic fuzzy soft set theory. Then based on some of these new matrix operations a new efficient methodology has been developed to solve intuitionistic fuzzy soft set based real life decision making problems which may contain more than one decision maker and an effort to apply it to a more relevant subject of todays world as in Predicting Terrorist Attack.

Keywords: Intuitionistic Fuzzy Soft Set, Intuitionistic Fuzzy Soft Matrix, Complement, Transpose, Choice Matrix, Symmetric Intuitionistic Fuzzy Soft Matrix, Addition, Subtraction, Product.

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1 Introduction

We require methods which allow some form or other flexible information processing capacity for dealing with real life ambiguous situations which we encounter in our day to day lives. Soft set theory [1, 2, 3] is generally used to solve such problems. Initially Molodtsov [1] presented soft set as a completely generic mathematical tool for modeling uncertainties in the year 1999. Since there is hardly any limitation in describing the objects, researchers simplify the decision making process by selecting the form of parameters they require and subsequently makes it more efficient in the absence of partial information. Maji et al. have done further research on soft set theory [2, 3]. Presence of vagueness demanded Fuzzy Soft Set (FSS) [4] to come into picture. But satisfactory evaluation of membership values is not always possible because of the insufficiency in the available information (besides the presence of vagueness) situation. Evaluation of non-membership values is also not always possible for the same reason and as a result there exists an indeterministic part upon which hesitation survives. Certainly fuzzy soft set theory is not suitable to solve such problems. In those situations Intuitionistic Fuzzy Soft Set theory (IFSS)[7] may be more applicable.

Moreover Maji et al. [2] utilize the thinking of attributes reduction in rough set theory to reduce parameters set of a soft set. Then the different researchers Chen et al.[6] in 2005 Zou et al. [5] and Kong et al. [8] in 2008 have worked on parameter reduction of soft set and fuzzy soft set theory. Furthermore, Zou et al. [11] have presented data analysis approaches of soft sets under incomplete information. These approaches presented in [11] are preferable for reflecting actual states of incomplete data in soft sets. But the applications of soft set theory generally solve problems with the help of rough sets or fuzzy soft sets. In 2010, Cagman et al. [10] introduced a new soft set based decision making method which selects a set of optimum elements from the alternatives. In the same year, the same authors [9] have proposed the definition of soft matrices which are representations of soft sets. This representation has several advantages. It is easy to store and manipulate matrices and hence the soft sets represented by them in a computer.

In this paper we have proposed the concept of intuitionistic fuzzy soft matrix. Then we have defined different types of intuitionistic fuzzy soft matrices with giving proper examples. Here we have also proposed the concept of choice matrix associated with an intuitionistic fuzzy soft set. Moreover we have introduced some operations on intuitionistic fuzzy soft matrices and choice matrices. Then based on some of these new matrix operations a new efficient solution procedure has been developed to solve intuitionistic fuzzy soft set based real life decision making problems which may contain more than one decision maker. The speciality of this new approach is that it may solve any intuitionistic fuzzy soft set based decision making problem involving large number of decision makers very easily and the computational procedure is also very simple. At last to realize this newly proposed methodology we apply it to Predict Terrorist Attack.

2 Preliminaries

2.1 Definition: [3]

Let U be an initial universe set and E be a set of parameters. Let P(U) denotes the power set of U. Let $A \subseteq E$. A pair (F_A, E) is called a **soft set** over U, where F_A is a mapping given by, $F_A : E \rightarrow P(U)$ such that $F_A(e) = \phi$ if $e \notin A$.

Here F_A is called approximate function of the soft set (F_A, E). The set $F_A(e)$ is called e-approximate value set which consists of related objects of the parameter $e \in E$. In other words, a soft set over U is a parameterized family of subsets of the universe U.

Example 2.1

Let U be the set of four dresses, given by, U = $\{d_1, d_2, d_3, d_4\}$. Let E be the set of parameters, given by,

E = $\{\,costly, cheap, comfortable, beautiful, gorgeous\} = \{e_1, e_2, e_3, e_4, e_5\}$, where

e_1 stands for the parameter costly,

e_2 stands for the parameter cheap,

e_3 stands for the parameter comfortable,

e_4 stands for the parameter beautiful,

e_5 stands for the parameter gorgeous.

Let $A \subset E$, given by, $A = \{e_1, e_2, e_3, e_4\}$ Now suppose that, F_A is a mapping, defined as dresses(.) and given by, $F_A(e_1) = \{d_2, d_4\}, F_A(e_2) = \{d_1, d_3\}, F_A(e_3) = \{d_2, d_3\}$,

$F_A(e_4) = \{d_4\}$. Then the soft set

$(F_A, E) = \{\, costly\,dresses = \{d_2, d_4\},$

$cheap\ dresses = \{d_1, d_3\}, comfortable\ dresses = \{d_2, d_3\},$

$beautiful\ dresses = \{d_4\},$

$gorgeous\ dresses = \phi\}$

2.2 Definition: [4]

Let U be an initial universe set and E be a set of parameters(which are fuzzy words or sentences involving fuzzy words). Let P(U) denotes the set of all fuzzy sets of U. Let $A \subset E$. A pair (F_A, E) is called a **Fuzzy Soft Set** (FSS) over U, where F_A is a mapping given by, $F_A : E \to P(U)$ such that $F_A(e) = \tilde{\phi}$ if $e \notin A$ where $\tilde{\phi}$ is a null fuzzy set.

Example 2.2

Let U be the set of four cities, given by, U = $\{C_1, C_2, C_3, C_4\}$. Let E be the set of parameters (each parameter is a fuzzy word), given by, E = $\{\ highly, immensely, moderately, average, less\}$

$= \{e_1, e_2, e_3, e_4, e_5\}$, where

e_1 stands for the parameter highly,

e_2 stands for the parameter immensely,

e_3 stands for the parameter moderately,

e_4 stands for the parameter average.

e_5 stands for the parameter less.

Let $A \subset E$, given by, $A = \{e_1, e_2, e_3, e_5\}$, Now suppose that,

$F_A(e_1) = \{C_1/.2, C_2/.9, C_3/.4, C_4/.6\}$,

$F_A(e_2) = \{C_2/1, C_3/.3, C_4/.4\}$,

$F_A(e_3) = \{C_1/.3, C_2/.4, C_3/.8\}$,

$F_A(e_5) = \{C_1/.9, C_2/.1, C_3/.5, C_4/.3\}$.

Then the fuzzy soft set is given by,

$(F_A, E) = \{highly\ polluted\ city = \{C_1/.2, C_2/.9, C_3/.4, C_4/.6\},$

$immensely\ polluted\ city = \{C_2/1, C_3/.3, C_4/.4\},$

$moderately\ polluted\ city = \{C_1/.3, C_2/.4, C_3/.8\},$

$average\ polluted\ city = \tilde{\phi},$

$less\ polluted\ city = \{C_1/.9, C_2/.1, C_3/.5, C_4/.3\}\}$

2.3 Definition: [7]

Let U be an initial universe set and E be a set of parameters. Let P(U) denotes the set of all intuitionistic fuzzy sets of U. Let $A \subset E$. A pair $(\hat{F}_A, E)$ is called an **Intuitionistic Fuzzy Soft Set**(IFSS) over U, where $\hat{F}_A$ is a mapping given by, $\hat{F}_A : E \to P(U)$ s.t, $\hat{F}_A(e) = \hat{\phi}$ if $e \notin A$ where $\hat{\phi}$ is null intuitionistic fuzzy set(ie. the membership value of x, $\mu(x) = 0$; the non-membership value of x, $\nu(x) = 1$ and the indeterministic part of x, $\pi(x) = 0 \forall x \in \hat{\phi}$).

Example 2.3

Let U be the set of four cities, given by, U = $\{C_1, C_2, C_3, C_4\}$.

Let E be the set of parameters (each parameter is a intuitionistic fuzzy word), given by,

E = $\{\ highly, immensely, moderately, average, less\ \} = \{e_1, e_2, e_3, e_4, e_5\}$ where

e_1 stands for the parameter highly,

e_2 stands for the parameter immensely,

e_3 stands for the parameter moderately,

e_4 stands for the parameter average,

e_5 stands for the parameter less.

Let $A \subset E$, given by,

A = $\{e_1, e_2, e_3, e_5\}$

Now suppose that, F_A is a mapping, defined as polluted cities(.) and given by,

$\hat{F}_A(e_1) = \{C_1/(.2,.7), C_2/(.8,.1), C_3/(.4,.2), C_4/(.6,.3)\},$

$\hat{F}_A(e_2) = \{C_2/(.9,.1), C_3/(.3,.6), C_4/(.4,.6)\},$

$\hat{F}_A(e_3) = \{C_1/(.3,.5), C_2/(.4,.6), C_3/(.8,.1)\},$

$\hat{F}_A(e_5) = \{C_1/(.9,.1), C_2/(.1,.8), C_3/(.5,.4), C_4/(.3,.5)\}$

Then the Intuitionistic Fuzzy Soft Set

$$(\hat{F}_A, E) = \{highly\ polluted\ city = \{C_1/(.2,.7), C_2/(.8,.1), C_3/(.4,.2), C_4/(.6,.3)\},$$

$$immensely\ polluted\ city = \{C_2/(.9,.1), C_3/(.3,.6), C_4/(.4,.6)\},$$

$$moderately\ polluted\ city = \{C_1/(.3,.5), C_2/(.4,.6), C_3/(.8,.1)\},$$

$$average\ polluted\ city = \hat{\phi},$$

$$less\ polluted\ city = \{C_1/(.9,.1), C_2/(.1,.8), C_3/(.5,.4), C_4/(.3,.5)\}\}$$

2.4 Definition: [9]

Let (F_A, E) be a soft set over U. Then a subset of $U \times E$ is uniquely defined by

$$R_A = \{(u,e) : e \in A, u \in F_A(e)\}$$

which is called a relation form of (F_A, E). Now the **characteristic function** of R_A is written by,

$$\chi_{R_A} : U \times E \to \{0,1\},\ \chi_{R_A} = \begin{cases} 1, (u,e) \in R_A \\ 0, (u,e) \notin R_A \end{cases}$$

Let $U = \{u_1, u_2, \ldots\ldots, u_m\}$, $E = \{e_1, e_2, \ldots\ldots, e_n\}$, then R_A can be presented by a table as in the following form

	e_1	e_2	....	e_n
u_1	$\chi_{R_A}(u_1,e_1)$	$\chi_{R_A}(u_1,e_2)$	....	$\chi_{R_A}(u_1,e_n)$
u_2	$\chi_{R_A}(u_2,e_1)$	$\chi_{R_A}(u_2,e_2)$	....	$\chi_{R_A}(u_2,e_n)$
....	....	....	....	....
u_m	$\chi_{R_A}(u_m,e_1)$	$\chi_{R_A}(u_m,e_2)$	....	$\chi_{R_A}(u_m,e_n)$

If $a_{ij} = \chi_{R_A}(u_i, e_j)$, we can define a matrix

$$[a_{ij}]_{m\times n} = \begin{pmatrix} a_{11} & a_{12} & \dots\dots\dots\dots\dots\dots. & a_{1n} \\ a_{21} & a_{22} & \dots\dots\dots\dots\dots\dots\dots & a_{2n} \\ \dots\dots & \dots\dots & \dots\dots\dots\dots\dots\dots\dots & \dots\dots \\ a_{m1} & a_{m2} & \dots\dots\dots\dots\dots\dots\dots & a_{mn} \end{pmatrix}$$

which is called a **soft matrix** of order $m\times n$ corresponding to the soft set (F_A, E) over U. A soft set (F_A, E) is uniquely characterized by the matrix $[a_{ij}]_{m\times n}$. Therefore we shall identify any soft set with its soft matrix and use these two concepts as interchangeable.

Example 2.4

Assume that $U = \{u_1, u_2, u_3, u_4, u_5\}$ be a universal set and $E = \{e_1, e_2, e_3, e_4, e_5\}$ be a set of all parameters. If $A = \{e_2, e_3, e_4\}$ and $F_A(e_2) = \{u_2, u_4\}, F_A(e_3) = \phi, F_A(e_4) = U$, then we write a soft set

$(F_A, E) = \{(e_2, \{u_2, u_4\}), (e_4, U)\}$

and then the relation form of (F_A, E) is written by,

$R_A = \{(u_2, e_2), (u_4, e_2), (u_1, e_4), (u_2, e_4), (u_3, e_4), (u_4, e_4), (u_5, e_4)\}$.

Hence the soft matrix (a_{ij}) is written by,

$$(a_{ij}) = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

3 Some New Concepts of Matrices in Intuitionistic Fuzzy Soft Set Theory:

3.1 Intuitionistic Fuzzy Soft Matrix:

Let $(\hat{F}_A, E)$ be an intuitionistic fuzzy soft set over U. Then a subset of $U\times E$ is uniquely defined by

$$R_A = \{(u,e) : e \in A, u \in \hat{F}_A(e)\}$$

which is called a relation form of $(\hat{F}_A, E)$. Now the relation R_A is characterized by the membership function $\mu_A : U \times E \to [0,1]$ and the non-membership function $\nu_A : U \times E \to [0,1]$

Now if the set of universe $U = \{u_1, u_2, ...\ldots, u_m\}$ and the set of parameters $E = \{e_1, e_2, ..., e_n\}$, then R_A can be presented by a table as in the following form

	e_1	e_2	....	e_n
u_1	(μ_{A11}, ν_{A11})	(μ_{A12}, ν_{A12})	....	(μ_{A1n}, ν_{A1n})
u_2	(μ_{A21}, ν_{A21})	(μ_{A22}, ν_{A22})	....	(μ_{A2n}, ν_{A2n})
....	....	....	....	....
u_m	(μ_{Am1}, ν_{Am1})	(μ_{Am2}, ν_{Am2})	....	(μ_{Amn}, ν_{Amn})

where $(\mu_{Amn}, \nu_{Amn}) = (\mu_A(u_m, e_n), \nu_A(u_m, e_n))$

If $a_{ij} = (\mu_{Aij}, \nu_{Aij})$, we can define a matrix

$$(\hat{a}_{ij})_{m\times n} = \begin{pmatrix} a_{11} & a_{12} & \ldots\ldots\ldots\ldots\ldots\ldots. & a_{1n} \\ a_{21} & a_{22} & \ldots\ldots\ldots\ldots\ldots\ldots\ldots & a_{2n} \\ \ldots\ldots & \ldots\ldots & \ldots\ldots\ldots\ldots\ldots\ldots\ldots & \ldots\ldots \\ a_{m1} & a_{m2} & \ldots\ldots\ldots\ldots\ldots\ldots\ldots & a_{mn} \end{pmatrix}$$

which is called an **intuitionistic fuzzy soft matrix** of order $m \times n$ corresponding to the intuitionistic fuzzy soft set $(\hat{F}_A, E)$ over U. An intuitionistic fuzzy soft set $(\hat{F}_A, E)$ is uniquely characterized by the matrix $(\hat{a}_{ij})_{m\times n}$. Therefore we shall identify any intuitionistic fuzzy soft set with its intuitionistic fuzzy soft matrix and use these two concepts as interchangeable.

Example 3.1

Let U be the set of four cities, given by, $U = \{C_1, C_2, C_3, C_4, C_5\}$.

Let E be the set of parameters (each parameter is a intuitionistic fuzzy word), given by, E = $\{$ *highly, immensely, moderately, average, less* $\} = \{e_1, e_2, e_3, e_4, e_5\}$ (say)

Let $A \subset E$, given by,

$A = \{e_1, e_2, e_3, e_5\}$ (say)

Now suppose that, $\hat{F}_A$ is a mapping, defined as polluted cities(.) and given by,

$\hat{F}_A(e_1) = \{C_1/(.2,.7), C_2/(.8,.1), C_3/(.4,.2), C_4/(.6,.3), C_5/(.7,.2)\}$,

$\hat{F}_A(e_2) = \{C_1/(0,1), C_2/(.9,.1), C_3/(.3,.6), C_4/(.4,.6), C_5/(.6,.3)\}$,

$\hat{F}_A(e_3) = \{C_1/(.3,.5), C_2/(.4,.6), C_3/(.8,.1), C_4/(.1,.8), C_5/(.3,.7)\}$,

$\hat{F}_A(e_5) = \{C_1/(.9,.1), C_2/(.1,.8), C_3/(.5,.4), C_4/(.3,.5), C_5/(.1,.8)\}$

Then the Intuitionistic Fuzzy Soft Set

$$\begin{aligned}(\hat{F}_A, E) = \{&\textit{highly polluted city} = \{C_1/(.2,.7), C_2/(.8,.1), C_3/(.4,.2), C_4/(.6,.3), C_5/(.7,.2)\},\\ &\textit{immensely polluted city} = \{C_1/(0,.9), C_2/(.9,.1), C_3/(.3,.6), C_4/(.4,.6), C_5/(.6,.3)\},\\ &\textit{moderately polluted city} = \{C_1/(.3,.5), C_2/(.4,.6), C_3/(.8,.1), C_4/(.1,.8), C_5/(.3,.7)\},\\ &\textit{less polluted city} = \{C_1/(.9,.1), C_2/(.1,.8), C_3/(.5,.4), C_4/(.3,.5), C_5/(.1,.8)\}\}\end{aligned}$$

Therefore the relation form of $(\hat{F}_A, E)$ is written by,

$$\begin{aligned}R_A = \{&(\{C_1/(.2,.7), C_2/(.8,.1), C_3/(.4,.2), C_4/(.6,.3), C_5/(.7,.2)\}, e_1), (\{C_1/(0,.9), C_2/(.9,.1), C_3/(.3,.6),\\ &C_4/(.4,.6), C_5/(.6,.3)\}, e_2), (\{C_1/(.3,.5), C_2/(.4,.6), C_3/(.8,.1), C_4/(.1,.8), C_5/(.3,.7)\}, e_3),\\ &(\{C_1/(.9,.1), C_2/(.1,.8), C_3/(.5,.4), C_4/(.3,.5), C_5/(.1,.8)\}, e_5)\}\end{aligned}$$

Hence the intuitionistic fuzzy soft matrix $(\hat{a}_{ij})$ is written by,

$$(\hat{a}_{ij}) = \begin{pmatrix} (.2,.7) & (0,.9) & (.3,.5) & (0,1) & (.9,.1) \\ (.8,.1) & (.9,.1) & (.4,.6) & (0,1) & (.1,.8) \\ (.4,.2) & (.3,.6) & (.8,.1) & (0,1) & (.5,.4) \\ (.6,.3) & (.4,.6) & (.1,.8) & (0,1) & (.3,.5) \\ (.7,.2) & (.6,.3) & (.3,.7) & (0,1) & (.1,.8) \end{pmatrix}$$

3.2 Row-Intuitionistic Fuzzy Soft Matrix:

An intuitionistic fuzzy soft matrix of order $1\times n$ i.e., with a single row is called a **row-intuitionistic fuzzy soft matrix**. Physically, a row-intuitionistic fuzzy soft matrix formally corresponds to an intuitionistic fuzzy soft set whose universal set contains only one object.

Example 3.2

Suppose the universe set U contains only one dress d_1 and parameter set

E = { costly, beautiful, cheap, comfortable }= $\{e_1,e_2,e_3,e_4\}$. Let $A=\{e_2,e_3,e_4\}\subset E$ and $\hat{F}_A(e_2)=\{d_1/(.8,.1)\},\hat{F}_A(e_3)=\{d_1/(.3,.7)\},\hat{F}_A(e_4)=\{d_1/(.6,.3)\}$. Then we write an intuitionistic fuzzy soft set as

$$(\hat{F}_A,E)=\{(e_2,\{d_1/(.8,.1)\}),(e_3,\{d_1/(.3,.7)\}),(e_4,\{d_1/(.6,.3)\})\}$$

and then the relation form of $(\hat{F}_A,E)$ is written by,

$$R_A=\{(\{d_1/(.8,.1)\},e_2),(\{d_1/(.3,.7)\},e_3),(\{d_1/(.6,.3)\},e_4)\}$$

Hence the intuitionistic fuzzy soft matrix $(\hat{a}_{ij})$ is written by, $(\hat{a}_{ij})=\begin{pmatrix}(0,1) & (.8,.1) & (.3,.7) & (.6,.3)\end{pmatrix}$ which contains a single row and so it is a row-intuitionistic fuzzy soft matrix.

3.3 Column-Intuitionistic Fuzzy Soft Matrix:

An intuitionistic fuzzy soft matrix of order $m\times 1$ i.e., with a single column is called a **column-intuitionistic fuzzy soft matrix**. Physically, a column-intuitionistic fuzzy soft matrix formally corresponds to an intuitionistic fuzzy soft set whose parameter set contains only one parameter.

Example 3.3

Suppose the initial universe set U contains four dresses d_1,d_2,d_3,d_4 and the parameter set E contains only one parameter given by, $E=\{\,beautiful\,\}=\{e_1\}$. $\hat{F}:E\to P(U)$ s.t, $\hat{F}(e_1)=\{d_1/(0.7,0.2),d_2/(0.2,0.6),d_3/(0.8,0.1),d_4/(0.4,0.6)\}$. Then we write an intuitionistic fuzzy soft set

$$(\hat{F},E)=\{(e_1,\{d_1/(0.7,0.2),d_2/(0.2,0.6),d_3/(0.8,0.1),d_4/(0.4,0.6)\})\}$$

and then the relation form of $(\hat{F},E)$ is written by,

$R_E = \{(\{d_1/(0.7,0.2), d_2/(0.2,0.6), d_3/(0.8,0.1), d_4/(0.4,0.6)\}, e_1)\}$

Hence the intuitionistic fuzzy soft matrix $(\hat{a}_{ij})$ is written by, $(\hat{a}_{ij}) = \begin{pmatrix} (0.7,0.2) \\ (0.2,0.6) \\ (0.8,0.1) \\ (0.4,0.6) \end{pmatrix}$ which contains a single column and so it is an example of column-intuitionistic fuzzy soft matrix.

3.4 Square Intuitionistic Fuzzy Soft Matrix:

An intuitionistic fuzzy soft matrix of order $m \times n$ is said to be a **square intuitionistic fuzzy soft matrix** if $m = n$ ie., the number of rows and the number of columns are equal. That means a square-intuitionistic fuzzy soft matrix is formally equal to an intuitionistic fuzzy soft set having the same number of objects and parameters.

Example 3.4

Consider the example 3.1

Here since the intuitionistic fuzzy soft matrix $(\hat{a}_{ij})$ contains five rows and five columns, so it is a square-intuitionistic fuzzy soft matrix.

3.5 Complement of an intuitionistic fuzzy soft matrix:

Let $(\hat{a}_{ij})$ be an $m \times n$ intuitionistic fuzzy soft matrix, where $\hat{a}_{ij} = (\mu_{ij}, \nu_{ij}) \forall i, j$. Then the **complement** of $(\hat{a}_{ij})$ is denoted by $(\hat{a}_{ij})^o$ and is defined by,

$(\hat{a}_{ij})^o = (\hat{c}_{ij})$, where $(\hat{c}_{ij})$ is also an intuitionistic fuzzy soft matrix of order $m \times n$ and $\hat{c}_{ij} = (\nu_{ij}, \mu_{ij}) \forall i, j$.

Example 3.5

Consider the example 3.1

Then the complement of $(\hat{a}_{ij})$ is,

$$(\hat{a}_{ij})^o = \begin{pmatrix} (.7,.2) & (.9,0) & (.5,.3) & (1,0) & (.1,.9) \\ (.1,.8) & (.1,.9) & (.6,.4) & (1,0) & (.8,.1) \\ (.2,.4) & (.6,.3) & (.1,.8) & (1,0) & (.4,.5) \\ (.3,.6) & (.6,.4) & (.9,.1) & (1,0) & (.5,.3) \\ (.2,.7) & (.3,.6) & (.7,.3) & (1,0) & (.8,.1) \end{pmatrix}$$

3.6 Null Intuitionistic Fuzzy Soft Matrix:

An intuitionistic fuzzy soft matrix of order $m \times n$ is said to be a **null intuitionistic fuzzy soft matrix or zero intuitionistic fuzzy soft matrix** if all of its elements are $(0,1)$. A null intuitionistic fuzzy soft matrix is denoted by, $\hat{\Phi}$. Now the intuitionistic fuzzy soft set associated with a null intuitionistic fuzzy soft matrix must be a null intuitionistic fuzzy soft set.

Example 3.6

Let there are four dresses in the universe set U given by, $U = \{d_1, d_2, d_3, d_4\}$ and the parameter set E = { beautiful, cheap, comfortable, gorgeous }= $\{e_1, e_2, e_3, e_4\}$.

Let $A = \{e_1, e_2, e_3\} \subset E$. Now let $\hat{F}_A : E \to P(U)$ s.t,

$\hat{F}_A(e_1) = \{d_1/(0,1), d_2/(0,1), d_3/(0,1), d_4/(0,1)\} = \hat{\phi}, \hat{F}_A(e_2) = \hat{\phi}, \hat{F}_A(e_3) = \hat{\phi}$.

Then the intuitionistic fuzzy soft set

$(\hat{F}_A, E) = \{(e_1, \hat{\phi}), (e_2, \hat{\phi}), (e_3, \hat{\phi})\}$

and then the relation form of $(\hat{F}_A, E)$ is written by,

$R_A = \{(\hat{\phi}, e_1), (\hat{\phi}, e_2), (\hat{\phi}, e_3)\}$

Hence the intuitionistic fuzzy soft matrix $(\hat{a}_{ij})$ is written by,

$$(\hat{a}_{ij}) = \begin{pmatrix} (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) \end{pmatrix} = \hat{\Phi}$$

3.7 Complete or Absolute Intuitionistic Fuzzy Soft Matrix:

An intuitionistic fuzzy soft matrix of order $m \times n$ is said to be a **complete or absolute intuitionistic fuzzy soft matrix** if all of its elements are $(1,0)$. A complete or absolute intuitionistic fuzzy soft matrix is denoted by, C_A. Now the intuitionistic fuzzy soft set associated with an absolute intuitionistic fuzzy soft matrix must be an absolute intuitionistic fuzzy soft set.

Example 3.7 Let there are four dresses in the universe set U given by, $U = \{d_1, d_2, d_3, d_4\}$ and the parameter set E = { beautiful, cheap, comfortable, gorgeous }= $\{e_1, e_2, e_3, e_4\}$. Let

$A=\{e_1,e_2,e_3,e_4\}\subseteq E$ and $\hat{F}_A:E\to P(U)$ s.t,

$$\hat{F}_A(e_1)=\{d_1/(1,0),d_2/(1,0),d_3/(1,0),d_4/(1,0)\},\hat{F}_A(e_2)=\{d_1/(1,0),d_2/(1,0),d_3/(1,0),d_4/(1,0)\},$$

$$\hat{F}_A(e_3)=\{d_1/(1,0),d_2/(1,0),d_3/(1,0),d_4/(1,0)\},\hat{F}_A(e_4)=\{d_1/(1,0),d_2/(1,0),d_3/(1,0),d_4/(1,0)\}$$

.

Then the intuitionistic fuzzy soft set

$$(\hat{F}_A,E)=\{(e_1,\{d_1/(1,0),d_2/(1,0),d_3/(1,0),d_4/(1,0)\}),(e_2,\{d_1/(1,0),d_2/(1,0),d_3/(1,0),d_4/(1,0)\}),$$

$$(e_3,\{d_1/(1,0),d_2/(1,0),d_3/(1,0),d_4/(1,0)\}),(e_4,\{d_1/(1,0),d_2/(1,0),d_3/(1,0),d_4/(1,0)\})\}$$

and then the relation form of $(\hat{F}_A,E)$ is written by,

$$R_A=\{(\{d_1/(1,0),d_2/(1,0),d_3/(1,0),d_4/(1,0)\},e_1),(\{d_1/(1,0),d_2/(1,0),d_3/(1,0),d_4/(1,0)\},e_2),$$

$$(\{d_1/(1,0),d_2/(1,0),d_3/(1,0),d_4/(1,0)\},e_3),(\{d_1/(1,0),d_2/(1,0),d_3/(1,0),d_4/(1,0)\},e_4)\}$$

Hence the intuitionistic fuzzy soft matrix $(\hat{a}_{ij})$ is written by,

$$(\hat{a}_{ij})=\begin{pmatrix}(1,0)&(1,0)&(1,0)&(1,0)\\(1,0)&(1,0)&(1,0)&(1,0)\\(1,0)&(1,0)&(1,0)&(1,0)\\(1,0)&(1,0)&(1,0)&(1,0)\end{pmatrix}=C_A$$

3.8 Diagonal Intuitionistic Fuzzy Soft Matrix:

A square intuitionistic fuzzy soft matrix of order $m\times n$ is said to be a **diagonal-intuitionistic fuzzy soft matrix** if all of its non-diagonal elements are $(0,1)$.

Example 3.8

Suppose the initial universe set $U=\{d_1,d_2,d_3,d_4,d_5\}$ and the parameter set $E=\{e_1,e_2,e_3,e_4,e_5\}$. Let $\hat{F}:E\to P(U)$ s.t,

$$\hat{F}(e_1)=\{d_1/(0.8,0.1),d_2/(0,1),d_3/(0,1),d_4/(0,1),d_5/(0,1)\},$$

$$\hat{F}(e_2)=\{d_1/(0,1),d_2/(0.3,0.7),d_3/(0,1),d_4/(0,1),d_5/(0,1)\},$$

$$\hat{F}(e_3)=\{d_1/(0,1),d_2/(0,1),d_3/(1,0),d_4/(0,1),d_5/(0,1)\},$$

$$\hat{F}(e_4)=\{d_1/(0,1),d_2/(0,1),d_3/(0,1),d_4/(0.7,0.2),d_5/(0,1)\},$$

$\hat{F}(e_5) = \{d_1/(0,1), d_2/(0,1), d_3/(0,1), d_4/(0,1), d_5/(0.6,0.3)\}$.

Then the intuitionistic fuzzy soft set

$(\hat{F}, E) = \{(e_1, \{d_1/(0.8,0.1), d_2/(0,1), d_3/(0,1), d_4/(0,1), d_5/(0,1)\}), (e_2, \{d_1/(0,1), d_2/(0.3,0.7),$

$d_3/(0,1), d_4/(0,1), d_5/(0,1)\}), (e_3, \{d_1/(0,1), d_2/(0,1), d_3/(1,0), d_4/(0,1), d_5/(0,1)\}), (e_4, \{d_1/(0,1),$

$d_2/(0,1), d_3/(0,1), d_4/(0.7,0.2), d_5/(0,1)\}), (e_5, \{d_1/(0,1), d_2/(0,1), d_3/(0,1), d_4/(0,1), d_5/(0.6,0.3)\})\}$

and then the relation form of $(\hat{F}, E)$ is written by,

$R_A = \{(\{(e_1, \{d_1/(0.8,0.1), d_2/(0,1), d_3/(0,1), d_4/(0,1), d_5/(0,1)\}, e_1), (\{d_1/(0,1), d_2/(0.3,0.7),$

$d_3/(0,1), d_4/(0,1), d_5/(0,1)\}, e_2), (\{d_1/(0,1), d_2/(0,1), d_3/(1,0), d_4/(0,1), d_5/(0,1)\}, e_3), (\{d_1/(0,1),$

$d_2/(0,1), d_3/(0,1), d_4/(0.7,0.2), d_5/(0,1)\}, e_4), (\{d_1/(0,1), d_2/(0,1), d_3/(0,1), d_4/(0,1), d_5/(0.6,0.3)\}, e_5)\}$

Hence the intuitionistic fuzzy soft matrix $(\hat{a}_{ij})$ is written by,

$$(\hat{a}_{ij}) = \begin{pmatrix} (0.8,0.1) & (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0.3,0.7) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (1,0) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0.7,0.2) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) & (0.6,0.3) \end{pmatrix}$$ whose all non-diagonal elements are $(0,1)$ and so it is a diagonal-intuitionistic fuzzy soft matrix.

3.9 Transpose of an Intuitionistic Fuzzy Soft Matrix:

The **transpose** of a square intuitionistic fuzzy soft matrix $(\hat{a}_{ij})$ of order $m \times n$ is another square intuitionistic fuzzy soft matrix of order $n \times m$ obtained from $(\hat{a}_{ij})$ by interchanging its rows and columns. It is denoted by $(\hat{a}_{ij})^T$. Therefore the intuitionistic fuzzy soft set associated with $(\hat{a}_{ij})^T$ becomes a new intuitionistic fuzzy soft set over the same universe and over the same set of parameters.

Example 3.9

Consider the example 3.1.

Here $(\hat{F}_A, E)$ be an intuitionistic fuzzy soft set over the universe U and over the set of parameters E, given by,

$$(\hat{F}_A, E) = \{highly\ polluted\ city = \{C_1/(.2,.7), C_2/(.8,.1), C_3/(.4,.2), C_4/(.6,.3), C_5/(.7,.2)\},$$
$$immensely\ polluted\ city = \{C_1/(0,.9), C_2/(.9,.1), C_3/(.3,.6), C_4/(.4,.6), C_5/(.6,.3)\},$$
$$moderately\ polluted\ city = \{C_1/(.3,.5), C_2/(.4,.6), C_3/(.8,.1), C_4/(.1,.8), C_5/(.3,.7)\},$$
$$less\ polluted\ city = \{C_1/(.9,.1), C_2/(.1,.8), C_3/(.5,.4), C_4/(.3,.5), C_5/(.1,.8)\}\}$$

whose associated intuitionistic fuzzy soft matrix is,

$$(\hat{a}_{ij}) = \begin{pmatrix} (.2,.7) & (0,.9) & (.3,.5) & (0,1) & (.9,.1) \\ (.8,.1) & (.9,.1) & (.4,.6) & (0,1) & (.1,.8) \\ (.4,.2) & (.3,.6) & (.8,.1) & (0,1) & (.5,.4) \\ (.6,.3) & (.4,.6) & (.1,.8) & (0,1) & (.3,.5) \\ (.7,.2) & (.6,.3) & (.3,.7) & (0,1) & (.1,.8) \end{pmatrix}$$

Now its transpose intuitionistic fuzzy soft matrix is,

$$(\hat{a}_{ij})^T = \begin{pmatrix} (.2,.7) & (.8,.1) & (.4,.2) & (.6,.3) & (.7,.2) \\ (0,.9) & (.9,.1) & (.3,.6) & (.4,.6) & (.6,.3) \\ (.3,.5) & (.4,.6) & (.8,.1) & (.1,.8) & (.3,.7) \\ (0,1) & (0,1) & (0,1) & (0,1) & (0,1) \\ (.9,.1) & (.1,.8) & (.5,.4) & (.3,.5) & (.1,.8) \end{pmatrix}$$

Therefore the intuitionistic fuzzy soft set associated with $(\hat{a}_{ij})^T$ is,

$$(\hat{G}_A, E) = \{\ highly\ polluted\ city = \{C_1/(.2,.7), C_2/(0,.9), C_3/(.3,.5), C_4/(0,1), C_5/(.9,.1)\},$$
$$immensely\ polluted\ city = \{C_1/(.8,.1), C_2/(.9,.1), C_3/(.4,.6), C_4/(0,1), C_5/(.1,.8)\},$$
$$moderately\ polluted\ city = \{C_1/(.4,.2), C_2/(.3,.6), C_3/(.8,.1), C_4/(0,1), C_5/(.3,.7)\},$$
$$average\ polluted\ city = \{C_1/(.6,.3), C_2/(.4,.6), C_3/(.1,.8), C_4/(0,1), C_5/(.3,.5)\},$$
$$less\ polluted\ city = \{C_1/(.7,.2), C_2/(.6,.3), C_3/(.3,.7), C_4/(0,1), C_5/(.1,.8)\}\}$$

3.10 Choice Matrix:

It is a square matrix whose rows and columns both indicate parameters. If ξ is a choice matrix, then its element $\xi(i, j)$ is defined as follows:

$$\xi(i,j) = (1,0) \textit{ when } i-th \textit{ and } j-th \textit{ parameters are both choice parameters of the decision makers}$$

$$= (0,1) \textit{ otherwise, i.e. when atleast one of the } i-th \textit{ or } j-th \textit{ parameters be not under choice}$$

There are different types of choice matrices according to the number of decision makers. We may realize this by the following example.

Example 3.10

Suppose that U be a set of four factories, say,

U = $\{f_1, f_2, f_3, f_4\}$

Let E be a set of parameters, given by,

$E = \{\textit{costly, excellent work culture, assured production, good location, cheap}\}$ = $\{e_1, e_2, e_3, e_4, e_5\}$ (say)

Now let the intuitionistic fuzzy soft set $(\hat{F}, A)$ describing the quality of the factories, is given by,

$$(\hat{F}, E) = \{\textit{costly factories} = \{f_1/(0.9,0.1), f_2/(0.2,0.7), f_3/(0.4,0.5), f_4/(0.8,0.1)\},$$

$$\textit{factories with excellent work culture} = \{f_1/(0.8,0.1), f_2/(0.3,0.5), f_3/(0.5,0.4), f_4/(0.4,0.5)\},$$

$$\textit{factories with assured production} = \{f_1/(0.9,0.1), f_2/(0.2,0.7), f_3/(0.4,0.5), f_4/(0.8,0.1)\},$$

$$\textit{factories with good location} = \{f_1/(0.7,0.3), f_2/(0.9,0.1), f_3/(0.4,0.5), f_4/(0.8,0.2)\},$$

$$\textit{cheap factories} = \{f_1/(0.1,0.8), f_2/(0.7,0.1), f_3/(0.5,0.3), f_4/(0.2,0.7)\}\}$$

Suppose Mr.X wants to buy a factory on the basis of his choice parameters excellent work culture, assured production and cheap which form a subset P of the parameter set E.

Therefore $P = \{e_2, e_3, e_5\}$

Now **the choice matrix of Mr.X** is,

$$(\xi_{ij})_P = \begin{array}{cc} & e_P \\ e_P & \begin{pmatrix} (0,1) & (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (1,0) & (1,0) & (0,1) & (1,0) \\ (0,1) & (1,0) & (1,0) & (0,1) & (1,0) \\ (0,1) & (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (1,0) & (1,0) & (0,1) & (1,0) \end{pmatrix} \end{array}$$

Now suppose Mr.X and Mr.Y together wants to buy a factory according to their choice parameters. Let the choice parameter set of Mr.Y be,

$Q = \{e_1, e_2, e_3, e_4\}$

Then **the combined choice matrix of Mr.X and Mr.Y** is

$$(\xi_{ij})_{(P,Q)} = \begin{array}{cc} & e_Q \\ e_P & \begin{pmatrix} (0,1) & (0,1) & (0,1) & (0,1) & (0,1) \\ (1,0) & (1,0) & (1,0) & (1,0) & (0,1) \\ (1,0) & (1,0) & (1,0) & (1,0) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) & (0,1) \\ (1,0) & (1,0) & (1,0) & (1,0) & (0,1) \end{pmatrix} \end{array}$$

[Here the entries $e_{ij} = (1,0)$ indicates that e_i is a choice parameter of Mr.X and e_j is a choice parameter of Mr.Y. Now $e_{ij} = (0,1)$ indicates either e_i fails to be a choice parameter of Mr.X or e_j fails to be a choice parameter of Mr.Y.]

Again the above combined choice matrix of Mr.X and Mr.Y may be also presented in its transpose form as,

$$(\xi_{ij})_{(Q,P)} = \begin{array}{cc} & e_P \\ e_Q & \begin{pmatrix} (0,1) & (1,0) & (1,0) & (0,1) & (1,0) \\ (0,1) & (1,0) & (1,0) & (0,1) & (1,0) \\ (0,1) & (1,0) & (1,0) & (0,1) & (1,0) \\ (0,1) & (1,0) & (1,0) & (0,1) & (1,0) \\ (0,1) & (0,1) & (0,1) & (0,1) & (0,1) \end{pmatrix} \end{array}$$

Now let us see the form of the combined choice matrix associated with three decision makers. Suppose that Mr.Z is willing to buy a factory together with Mr.X and Mr.Y on the basis of his choice parameters excellent work culture, assured production and good location which form a subset R of the parameter set E.

Therefore $R = \{e_2, e_3, e_4\}$

Then **the combined choice matrix of Mr.X, Mr.Y and Mr.Z** will be of three different types which are as follows,

$$\text{i)}\ (\xi_{ij})_{(R,P\wedge Q)} = \begin{array}{cc} & e_{(P\wedge Q)} \\ e_R & \begin{pmatrix} (0,1) & (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (1,0) & (1,0) & (0,1) & (0,1) \\ (0,1) & (1,0) & (1,0) & (0,1) & (0,1) \\ (0,1) & (1,0) & (1,0) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) & (0,1) \end{pmatrix} \end{array}$$

[Since the set of common choice parameters of Mr.X and Mr.Y is, $P \wedge Q = \{e_2, e_3\}$. Here the entries $e_{ij} = (1,0)$ indicates that e_i is a choice parameter of Mr.Z and e_j is a common choice parameter of Mr.X and Mr.Y. Now $e_{ij} = (0,1)$ indicates either e_i fails to be a choice parameter of Mr.Z or e_j fails to be a common choice parameter of Mr.X and Mr.Y.]

$$\text{ii)}\ (\xi_{ij})_{(P,Q\wedge R)} = \begin{array}{cc} & e_{(Q\wedge R)} \\ e_P & \begin{pmatrix} (0,1) & (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (1,0) & (1,0) & (1,0) & (0,1) \\ (0,1) & (1,0) & (1,0) & (1,0) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (1,0) & (1,0) & (1,0) & (0,1) \end{pmatrix} \end{array} [\ Since\ Q \wedge R = \{e_2, e_3, e_4\}]$$

$$\text{iii)}\ (\xi_{ij})_{(Q,R\wedge P)} = \begin{array}{cc} & e_{(R\wedge P)} \\ e_Q & \begin{pmatrix} (0,1) & (1,0) & (1,0) & (0,1) & (0,1) \\ (0,1) & (1,0) & (1,0) & (0,1) & (0,1) \\ (0,1) & (1,0) & (1,0) & (0,1) & (0,1) \\ (0,1) & (1,0) & (1,0) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) & (0,1) \end{pmatrix} \end{array} [\ Since\ R \wedge P = \{e_2, e_3\}]$$

3.11 Symmetric Intuitionistic Fuzzy Soft Matrix:

A square intuitionistic fuzzy soft matrix A of order $n \times n$ is said to be a **symmetric Intuitionistic fuzzy soft matrix**, if its transpose be equal to it, i.e., if $A^T = A$. Hence the Intuitionistic fuzzy soft matrix $(\hat{a}_{ij})$ is symmetric, if $\hat{a}_{ij} = \hat{a}_{ji}, \forall i, j.$

Therefore if $(\hat{a}_{ij})$ be a symmetric Intuitionistic fuzzy soft matrix then the Intuitionistic fuzzy soft sets associated with $(\hat{a}_{ij})$ and $(\hat{a}_{ij})^T$ both be the same.

Example 3.11

Let the set of universe $U = \{u_1, u_2, u_3, u_4\}$ and the set of parameters $E = \{e_1, e_2, e_3, e_4\}$.

Now suppose that, $A \subseteq E$ and $\hat{F}_A : E \to P(U)$ s.t, $(\hat{F}_A, E)$ forms an intuitionistic fuzzy soft set given by,

$$(\hat{F}_A, E) = \{(e_1, \{u_1/(0.2,0.8), u_2/(0.3,0.7), u_3/(0.8,0.1), u_4/(0.5,0.4)\}), (e_2, \{u_1/(0.3,0.7), u_2/(0.6,0.2),$$

$$u_3/(0.1,0.7), u_4/(0.7,0.2)\}), (e_3, \{u_1/(0.8,0.1), u_2/(0.1,0.7), u_3/(0.7,0.2), u_4/(0.2,0.7)\}),$$

$$(e_4, \{u_1/(0.5,0.4), u_2/(0.7,0.2), u_3/(0.2,0.7), u_4/(0.4,0.6)\})\}$$

The intuitionistic fuzzy soft matrix associated with this intuitionistic fuzzy soft set $(\hat{F}_A, E)$ is,

$$(\hat{a}_{ij}) = \begin{pmatrix} (0.2,0.8) & (0.3,0.7) & (0.8,0.1) & (0.5,0.4) \\ (0.3,0.7) & (0.6,0.2) & (0.1,0.7) & (0.7,0.2) \\ (0.8,0.1) & (0.1,0.7) & (0.7,0.2) & (0.2,0.7) \\ (0.5,0.4) & (0.7,0.2) & (0.2,0.7) & (0.4,0.6) \end{pmatrix}$$

Since here $\hat{a}_{ij} = \hat{a}_{ji}, \forall i, j$; $(\hat{a}_{ij})$ is a symmetric intuitionistic fuzzy soft matrix.

3.12 Addition of Intuitionistic Fuzzy Soft Matrices: Two intuitionistic fuzzy soft matrices A and B are said to be conformable for addition, if they be of the same order. The addition of two intuitionistic fuzzy soft matrices $(\hat{a}_{ij})$ and $(\hat{b}_{ij})$ of order $m \times n$ is defined by,

$(\hat{a}_{ij}) \oplus (\hat{b}_{ij}) = (\hat{c}_{ij})$, where $(\hat{c}_{ij})$ is also an $m \times n$ intuitionistic fuzzy soft matrix and

$$\hat{c}_{ij} = (max\{\mu_{\hat{a}_{ij}}, \mu_{\hat{b}_{ij}}\}, min\{\nu_{\hat{a}_{ij}}, \nu_{\hat{b}_{ij}}\}) \forall i, j.$$

Example 3.12

Consider the intuitionistic fuzzy soft matrix of example 3.1,

$$(\hat{a}_{ij}) = \begin{pmatrix} (.2,.7) & (0,.9) & (.3,.5) & (0,1) & (.9,.1) \\ (.8,.1) & (.9,.1) & (.4,.6) & (0,1) & (.1,.8) \\ (.4,.2) & (.3,.6) & (.8,.1) & (0,1) & (.5,.4) \\ (.6,.3) & (.4,.6) & (.1,.8) & (0,1) & (.3,.5) \\ (.7,.2) & (.6,.3) & (.3,.7) & (0,1) & (.1,.8) \end{pmatrix}$$

Now consider another intuitionistic fuzzy soft matrix $(\hat{b}_{ij})$ associated with the intuitionistic fuzzy soft set $(\hat{G}_B, E)$ (also describing the pollution of the cities) over the same universe U.

Let $B = \{e_1, e_4, e_5\} \subset E$ and

$$(\hat{G}, B) = \{highly\ polluted\ city = \{C_1/(.3,.7), C_2/(.9,.1), C_3/(.4,.5), C_4/(.7,.2), C_5/(.6,.2)\},$$

$$average\ polluted\ city = \{C_1/(.2,.7), C_2/(.3,.7), C_3/(.7,.1), C_4/(.2,.8), C_5/(.3,.6)\},$$

$$less\ polluted\ city = \{C_1/(.8,.1), C_2/(.2,.7), C_3/(.6,.4), C_4/(.3,.5), C_5/(.2,.6)\}\}$$

and then the relation form of $(\hat{G}_B, E)$ is written by,

$$R_B = \{(\{C_1/(.3,.7), C_2/(.9,.1), C_3/(.4,.5), C_4/(.7,.2), C_5/(.6,.2)\}, e_1), (\{C_1/(.2,.7), C_2/(.3,.7),$$
$$C_3/(.7,.1), C_4/(.2,.8), C_5/(.3,.6)\}, e_2), (\{d_1/0.3, d_2/0.7, d_3/0.6, d_4/0.2\}, e_4),$$
$$(\{C_1/(.8,.1), C_2/(.2,.7), C_3/(.6,.4), C_4/(.3,.5), C_5/(.2,.6)\}, e_5)\}$$

Hence the intuitionistic fuzzy soft matrix $(\hat{b}_{ij})$ is written by,

$$(\hat{b}_{ij}) = \begin{pmatrix} (.3,.7) & (0,1) & (0,1) & (.2,.7) & (.8,.1) \\ (.9,.1) & (0,1) & (0,1) & (.3,.7) & (.2,.7) \\ (.4,.5) & (0,1) & (0,1) & (.7,.1) & (.6,.4) \\ (.7,.2) & (0,1) & (0,1) & (.2,.8) & (.3,.5) \\ (.6,.2) & (0,1) & (0,1) & (.3,.6) & (.2,.6) \end{pmatrix}$$

Therefore the sum of the intuitionistic fuzzy soft matrices $(\hat{a}_{ij})$ and $(\hat{b}_{ij})$ is,

$$(\hat{a}_{ij}) \oplus (\hat{b}_{ij}) = \begin{pmatrix} (.3,.7) & (0,0.9) & (.3,.5) & (0.2,0.7) & (.9,.1) \\ (.9,.1) & (.9,0.1) & (.4,0.6) & (0.3,.7) & (.2,.7) \\ (.4,.2) & (.3,0.6) & (.8,0.1) & (0.7,.1) & (.6,.4) \\ (.7,.2) & (.4,0.6) & (.1,0.8) & (0.2,.8) & (.3,.5) \\ (.7,.2) & (.6,0.3) & (.3,0.7) & (0.3,.6) & (.2,.6) \end{pmatrix}$$

3.13 Subtraction of Intuitionistic Fuzzy Soft Matrices: Two intuitionistic fuzzy soft matrices A and B are said to be conformable for subtraction, if they be of the same order. For any two intuitionistic fuzzy soft matrices $(\hat{a}_{ij})$ and $(\hat{b}_{ij})$ of order $m \times n$, the subtraction of $(\hat{b}_{ij})$ from $(\hat{a}_{ij})$ is defined as,

$(\hat{a}_{ij}) ! (\hat{b}_{ij}) = (\hat{c}_{ij})$, where $(\hat{c}_{ij})$ is also an $m \times n$ intuitionistic fuzzy soft matrix and

$c_{ij} = (min\{\mu_{\hat{a}_{ij}}, \mu_{\hat{b}^o_{ij}}\}, max\{\nu_{\hat{a}_{ij}}, \nu_{\hat{b}^o_{ij}}\}) \forall i, j$ where $(\hat{b}^o_{ij})$ is the complement of $(\hat{b}_{ij})$

Example 3.13

Consider the intuitionistic fuzzy soft matrices $(\hat{a}_{ij})$ and $(\hat{b}_{ij})$ of example 3.12

Now $$(\hat{a}_{ij}) = \begin{pmatrix} (.2,.7) & (0,.9) & (.3,.5) & (0,1) & (.9,.1) \\ (.8,.1) & (.9,.1) & (.4,.6) & (0,1) & (.1,.8) \\ (.4,.2) & (.3,.6) & (.8,.1) & (0,1) & (.5,.4) \\ (.6,.3) & (.4,.6) & (.1,.8) & (0,1) & (.3,.5) \\ (.7,.2) & (.6,.3) & (.3,.7) & (0,1) & (.1,.8) \end{pmatrix}$$

and $$(\hat{b}_{ij})^o = \begin{pmatrix} (.7,.3) & (1,0) & (1,0) & (.7,.2) & (.1,.8) \\ (.1,.9) & (1,0) & (1,0) & (.7,.3) & (.7,.2) \\ (.5,.4) & (1,0) & (1,0) & (.1,.7) & (.4,.6) \\ (.2,.7) & (1,0) & (1,0) & (.8,.2) & (.5,.3) \\ (.2,.6) & (1,0) & (1,0) & (.6,.3) & (.6,.2) \end{pmatrix}$$

Therefore the subtraction of the intuitionistic fuzzy soft matrix $(\hat{b}_{ij})$ from the intuitionistic fuzzy soft matrix $(\hat{a}_{ij})$ is,

$$(\hat{a}_{ij})!(\hat{b}_{ij}) = \begin{pmatrix} (.2,.7) & (0,.9) & (0.3,.5) & (0,0.2) & (.1,.8) \\ (.1,.9) & (0.9,.1) & (0.4,.6) & (0,0.3) & (.1,.8) \\ (.2,.7) & (0.3,.6) & (0.8,.1) & (0,0.7) & (.4,.6) \\ (.2,.7) & (0.4,.6) & (0.1,.8) & (0,0.2) & (.3,.5) \\ (.2,.6) & (0.6,.3) & (0.3,.7) & (0,0.3) & (.1,.8) \end{pmatrix}$$

3.14 Properties: Let A and B be two intuitionistic fuzzy soft matrices of order $m \times n$. Then

i) $A \oplus B = B \oplus A$

ii) $A ! B \neq B ! A$

iii) $A \oplus A^o \neq C_A$

iv) $A ! A \neq \hat{\Phi}$

3.15 Product of an intuitionistic Fuzzy Soft Matrix with a Choice Matrix: Let U be the set of universe and E be the set of parameters. Suppose that A be any intuitionistic fuzzy soft matrix and β be any choice matrix of a decision maker concerned with the same universe U and E. Now if the number of columns of the intuitionistic fuzzy soft matrix A be equal to the number of rows of the choice matrix β, then A and β are said to be conformable for the product ($A \otimes \beta$) and the product ($A \otimes \beta$) becomes an intuitionistic fuzzy soft matrix. We may denote the product by $A \otimes \beta$ or simply by $A\beta$.

If $A = (\hat{a}_{ij})_{m \times n}$ and $\beta = (\hat{\beta}_{jk})_{n \times p}$, then

$A\beta = (\hat{c}_{ik})$

where $\hat{c}_{ik} = (max_{j=1}^{n} min\{\mu_{\hat{a}_{ij}}, \mu_{\hat{\beta}_{jk}}\}, min_{j=1}^{n} max\{\nu_{\hat{a}_{ij}}, \nu_{\hat{\beta}_{jk}}\})$

It is to be noted that, βA cannot be defined here.

Example 3.14

Let U be the set of four dresses, given by, U = $\{d_1, d_2, d_3, d_4\}$. Let E be the set of parameters, given by, E = $\{cheap, beautiful, comfortable, gorgeous\} = \{e_1, e_2, e_3, e_4\}$ (say). Suppose that the set of choice parameters of Mr.X be, $A = \{e_1, e_3\}$. Now let according to the choice parameters of Mr.X, we have the intuitionistic fuzzy soft set $(\hat{F}, A)$ which describes the attractiveness of the dresses and the intuitionistic fuzzy soft matrix of the intuitionistic fuzzy soft set $(\hat{F}, A)$ be,

$$(\hat{a}_{ij}) = \begin{pmatrix} (0.8,0.1) & (0.2,0.7) & (0.7,0.2) & (0.3,0.5) \\ (0.3,0.6) & (0.7,0.1) & (0.4,0.6) & (0.8,0.1) \\ (0.7,0.2) & (0.4,0.5) & (0.5,0.3) & (0.6,0.2) \\ (0.5,0.4) & (0.1,0.8) & (0.9,0.1) & (0.2,0.7) \end{pmatrix}$$

Again the choice matrix of Mr.X is,

$$(\xi_{ij})_A = \begin{array}{c} \quad e_A \\ e_A \begin{pmatrix} (1,0) & (0,1) & (1,0) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) \\ (1,0) & (0,1) & (1,0) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) \end{pmatrix} \end{array}$$

Since the number of columns of the intuitionistic fuzzy soft matrix $(\hat{a}_{ij})$ is equal to the number of rows of the choice matrix $(\xi_{ij})_A$, they are conformable for the product.

$$\text{Therefore} \begin{pmatrix} (0.8,0.1) & (0.2,0.7) & (0.7,0.2) & (0.3,0.5) \\ (0.3,0.6) & (0.7,0.1) & (0.4,0.6) & (0.8,0.1) \\ (0.7,0.2) & (0.4,0.5) & (0.5,0.3) & (0.6,0.2) \\ (0.5,0.4) & (0.1,0.8) & (0.9,0.1) & (0.2,0.7) \end{pmatrix} \otimes \begin{pmatrix} (1,0) & (0,1) & (1,0) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) \\ (1,0) & (0,1) & (1,0) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) \end{pmatrix}$$

$$= \begin{pmatrix} (0.8,0.1) & (0,1) & (0.8,0.1) & (0,1) \\ (0.4,0.6) & (0,1) & (0.4,0.6) & (0,1) \\ (0.7,0.2) & (0,1) & (0.7,0.2) & (0,1) \\ (0.9,0.1) & (0,1) & (0.9,0.1) & (0,1) \end{pmatrix}$$

4 A New Approach to Solve Intuitionistic Fuzzy Soft Set Based Decision Making Problems:

This new approach is specially based on choice matrices. These choice matrices represent the choice parameters of the decision makers and also help us to solve the intuitionistic fuzzy soft matrix based decision making problems with least computational complexity. Now at first we consider a generalized intuitionistic fuzzy soft set based decision making problem.

4.1 A Generalized Intuitionistic Fuzzy Soft Set Based Decision Making Problem: Suppose that the initial universal set U be the set of m objects $O_1, O_2, \ldots\ldots, O_m$ and the set of parameters E be given by, $E = \{e_1, e_2, \ldots\ldots, e_n\}$ [where each parameter is an intuitionistic fuzzy word or sentences involving intuitionistic fuzzy words]. Now let N number of decision makers $D_1, D_2, \ldots., D_N$ want to select an object from U according to their set of choice parameters $P_{D_1}, P_{D_2}, \ldots., P_{D_N}$ [where $P_{D_1}, P_{D_2}, \ldots., P_{D_N} \subseteq E$] respectively. Now the problem is to find out the optimal object from U which satisfies all of these choice parameters of the decision makers as much as possible.

4.2 The Stepwise Solving Procedure: To solve such type of intuitionistic fuzzy soft set based decision making problems, we are presenting the following stepwise procedure which comprises of the newly proposed choice matrices and the operations on them.

Algorithm:

Step-I: First construct the combined choice matrix with respect to the choice parameters of the decision makers.

Step-II: Compute the product intuitionistic fuzzy soft matrices by multiplying each given intuitionistic fuzzy soft matrix with the combined choice matrix as per the rule of multiplication of intuitionistic fuzzy soft matrices.

Step-III: Compute the sum of these product intuitionistic fuzzy soft matrices to have the resultant intuitionistic fuzzy soft matrix(R_f).

Step-IV: Then compute the weight of each object(O_i) by adding the membership values of the entries of its concerned row(i-th row) of R_f and denote it as $W(O_i)$.

Step-V: The object having the highest weight becomes the optimal choice object. If more than one object have the highest weight then go to the next step.

Step-VI: Now we have to consider the sum of the non-membership values (Θ) of the entries of the rows associated with those equal weighted objects. The object with the minimum Θ -value will be the optimal choice object. Now if the Θ-values of those objects also be the same, any one of them may be chosen as the optimal choice object.

To illustrate the basic idea of the algorithm, now we apply it to the following intuitionistic fuzzy soft matrix based decision making problems.

Example 4.1

Let U be the set of four dresses, given by, U = $\{d_1, d_2, d_3, d_4\}$. Let E be the set of parameters, given by, E = $\{cheap, beautiful, comfortable, gorgeous\} = \{e_1, e_2, e_3, e_4\}$ (say). Suppose that, three friends Mr.X, Mr.Y and Mr.Z together want to buy a dress among these four dresses for their common friend Mr.D according to their choice parameters, $A = \{e_1, e_3\}, B = \{e_2, e_3\}, C = \{e_1, e_4\}$ respectively. Now let according to the choice parameters of Mr.X, Mr.Y and Mr.Z, we have the intuitionistic fuzzy soft sets (F_A, E), (G_B, E), (H_C, E) which describe the attractiveness of the dresses according to Mr.X, Mr.Y and Mr.Z respectively. Let the intuitionistic fuzzy soft matrices of the intuitionistic fuzzy soft sets $(F_A, E), (G_B, E)$ and (H_C, E) are respectively,

$$(a_{ij}) = \begin{pmatrix} (0.9,0.1) & (0,1) & (1,0) & (0,1) \\ (0.3,0.5) & (0,1) & (0.6,0.3) & (0,1) \\ (0.7,0.1) & (0,1) & (0.3,0.5) & (0,1) \\ (0.2,0.7) & (0,1) & (0.2,0.7) & (0,1) \end{pmatrix}, \quad (b_{ik}) = \begin{pmatrix} (0,1) & (0.4,0.6) & (0.8,0.1) & (0,1) \\ (0,1) & (0.8,0.1) & (0.6,0.2) & (0,1) \\ (0,1) & (0.5,0.2) & (0.4,0.5) & (0,1) \\ (0,1) & (0.3,0.5) & (0.2,0.8) & (0,1) \end{pmatrix},$$

$$(c_{il}) = \begin{pmatrix} (0.9,0.1) & (0,1) & (0,1) & (0.2,0.7) \\ (0.4,0.5) & (0,1) & (0,1) & (0.3,0.5) \\ (0.6,0.3) & (0,1) & (0,1) & (0.6,0.2) \\ (0.3,0.5) & (0,1) & (0,1) & (0.9,0) \end{pmatrix}$$

Now the problem is to select the dress among the four dresses which satisfies the choice parameters of Mr.X, Mr.Y and Mr.Z as much as possible.

1) The combined choice matrices of Mr.X, Mr.Y, Mr.Z in different forms are,

$$\begin{array}{cc} & e_{B\wedge C} \\ e_A & \begin{pmatrix} (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) \end{pmatrix} \end{array} \quad [\text{ Since } B \wedge C = \phi, A = \{e_1, e_3\}]$$

$$\begin{array}{cc} & e_{C\wedge A} \\ e_B & \begin{pmatrix} (0,1) & (0,1) & (0,1) & (0,1) \\ (1,0) & (0,1) & (0,1) & (0,1) \\ (1,0) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) \end{pmatrix} \end{array} \quad [\text{ Since } \ C\wedge A=\{e_1\}, B=\{e_2,e_3\}]$$

$$\begin{array}{cc} & e_{A\wedge B} \\ e_C & \begin{pmatrix} (0,1) & (0,1) & (1,0) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (1,0) & (0,1) \end{pmatrix} \end{array} \quad [\text{ Since } \ A\wedge B=\{e_3\}, C=\{e_1,e_4\}]$$

2) Corresponding product fuzzy soft matrices are,

$$\begin{array}{cc} & e_A \\ U_A & \begin{pmatrix} (0.9,0.1) & (0,1) & (1,0) & (0,1) \\ (0.3,0.5) & (0,1) & (0.6,0.3) & (0,1) \\ (0.7,0.1) & (0,1) & (0.3,0.5) & (0,1) \\ (0.2,0.7) & (0,1) & (0.2,0.7) & (0,1) \end{pmatrix} \end{array} \otimes \begin{array}{cc} & e_{B\wedge C} \\ e_A & \begin{pmatrix} (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) \end{pmatrix} \end{array}$$

$$= \begin{pmatrix} (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) \end{pmatrix}$$

$$\begin{array}{cc} & e_B \\ U_B & \begin{pmatrix} (0,1) & (0.4,0.6) & (0.8,0.1) & (0,1) \\ (0,1) & (0.8,0.1) & (0.6,0.2) & (0,1) \\ (0,1) & (0.5,0.2) & (0.4,0.5) & (0,1) \\ (0,1) & (0.3,0.5) & (0.2,0.8) & (0,1) \end{pmatrix} \end{array} \otimes \begin{array}{cc} & e_{C\wedge A} \\ e_B & \begin{pmatrix} (0,1) & (0,1) & (0,1) & (0,1) \\ (1,0) & (0,1) & (0,1) & (0,1) \\ (1,0) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) \end{pmatrix} \end{array}$$

$$= \begin{pmatrix} (0.8,0.1) & (0,1) & (0,1) & (0,1) \\ (0.8,0.1) & (0,1) & (0,1) & (0,1) \\ (0.5,0.2) & (0,1) & (0,1) & (0,1) \\ (0.3,0.5) & (0,1) & (0,1) & (0,1) \end{pmatrix}$$

$$\begin{array}{c} \\ U_C \end{array} \begin{array}{c} e_C \\ \begin{pmatrix} (0.9,0.1) & (0,1) & (0,1) & (0.2,0.7) \\ (0.4,0.5) & (0,1) & (0,1) & (0.3,0.5) \\ (0.6,0.3) & (0,1) & (0,1) & (0.6,0.2) \\ (0.3,0.5) & (0,1) & (0,1) & (0.9,0) \end{pmatrix} \end{array} \otimes \begin{array}{c} \\ e_C \end{array} \begin{array}{c} e_{A \wedge B} \\ \begin{pmatrix} (0,1) & (0,1) & (1,0) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (1,0) & (0,1) \end{pmatrix} \end{array}$$

$$= \begin{pmatrix} (0,1) & (0,1) & (0.9,0.1) & (0,1) \\ (0,1) & (0,1) & (0.4,0.5) & (0,1) \\ (0,1) & (0,1) & (0.6,0.2) & (0,1) \\ (0,1) & (0,1) & (0.9,0) & (0,1) \end{pmatrix}$$

[As per the rule of multiplication of fuzzy soft matrices.]

3) The sum of these product fuzzy soft matrices is,

$$\begin{pmatrix} (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) \end{pmatrix} \oplus \begin{pmatrix} (0.8,0.1) & (0,1) & (0,1) & (0,1) \\ (0.8,0.1) & (0,1) & (0,1) & (0,1) \\ (0.5,0.2) & (0,1) & (0,1) & (0,1) \\ (0.3,0.5) & (0,1) & (0,1) & (0,1) \end{pmatrix} \oplus \begin{pmatrix} (0,1) & (0,1) & (0.9,0.1) & (0,1) \\ (0,1) & (0,1) & (0.4,0.5) & (0,1) \\ (0,1) & (0,1) & (0.6,0.2) & (0,1) \\ (0,1) & (0,1) & (0.9,0) & (0,1) \end{pmatrix}$$

$$= \begin{pmatrix} (0.8,0.1) & (0,1) & (0.9,0.1) & (0,1) \\ (0.8,0.1) & (0,1) & (0.4,0.5) & (0,1) \\ (0.5,0.2) & (0,1) & (0.6,0.2) & (0,1) \\ (0.3,0.5) & (0,1) & (0.9,0) & (0,1) \end{pmatrix} = R_f$$

4) Now the weights of the dresses are,

- $W(d_1) = 0.8 + 0 + 0.9 + 0 = 1.7$
- $W(d_2) = 0.8 + 0 + 0.4 + 0 = 1.2$

- $W(d_3) = 0.5 + 0 + 0.6 + 0 = 1.1$
- $W(d_4) = 0.3 + 0 + 0.9 + 0 = 1.2$

5) The dress associated with the first row of the resultant fuzzy soft matrix(R_f) has the highest weight $(W(d_1) = 1.7)$, therefore d_1 be the optimal choice dress. Hence Mr.X, Mr.Y and Mr.Z will buy the dress d_1 according to their choice parameters.

5 Predicting Terrorist Attack

One of the worst enemy of the modern day civilization is terrorism. It has scaled new heights with the help of ever evolving modern day technology and communication system rendering the common mass vulnerable to the ceaseless act of violence. Now terrorism has many faces both from the national and international arena with a common goal to disrupt the very sense of peace and security of our daily lives. Among those we are here considering the three organizations - X, Y and Z for our example.

Example 5.1

Let U be the set of four cities, given by, $U = \{C_1, C_2, C_3, C_4\}$.

Let E be the set of parameters, given by,

E = $\{$ *high populationdensity, destruction of the government property, pre and post attack hiding, maximum media coverage* $\} = \{e_1, e_2, e_3, e_4\}$ (say).

Now suppose that - X, Y and Z be planned to attack a city in between the above four. All the groups X, Y and Z want to destroy government properties and seek maximum media coverage for their publicity. Now Maoists are more comfortable and specializes in forest attacks and they give consideration of pre and post attack hiding. In contrast, Laskar-e-taiba and Alkaida always plan to attack a city with high density population to kill as much people as possible. So the choice parameters of the gangs X, Y and Z are respectively,

$A = \{e_1, e_2, e_4\}, B = \{e_1, e_2, e_4\}$ and $C = \{e_2, e_3, e_4\}$. Now let the intuitionistic fuzzy soft matrices associated with the intuitionistic fuzzy soft sets $(\hat{F}_A, E), (\hat{G}_B, E)$ and $(\hat{H}_C, E)$ describing the importance of the cities to attack according to gang X, Y and Z be respectively,

$$(\hat{a}_{ij}) = \begin{pmatrix} (0.8,0.1) & (0.7,0.1) & (0,1) & (0.9,0.1) \\ (0.4,0.5) & (0.3,0.7) & (0,1) & (0.4,0.5) \\ (0.6,0.3) & (0.4,0.3) & (0,1) & (0.6,0.2) \\ (0.7,0.1) & (0.5,0.2) & (0,1) & (0.7,0.2) \end{pmatrix}$$

$$(\hat{b}_{ik}) = \begin{pmatrix} (0.7,0.2) & (0.6,0.1) & (0,1) & (0.8,0.1) \\ (0.3,0.5) & (0.3,0.7) & (0,1) & (0.4,0.5) \\ (0.5,0.3) & (0.5,0.3) & (0,1) & (0.6,0.4) \\ (0.6,0.2) & (0.4,0.2) & (0,1) & (0.6,0.2) \end{pmatrix}$$

$$(\hat{c}_{il}) = \begin{pmatrix} (0,1) & (0.8,0.1) & (0.2,0.7) & (0.9,0.1) \\ (0,1) & (0.3,0.7) & (0.9,0.1) & (0.4,0.5) \\ (0,1) & (0.5,0.2) & (0.6,0.3) & (0.7,0.1) \\ (0,1) & (0.6,0.2) & (0.4,0.5) & (0.6,0.2) \end{pmatrix}$$

Now the problem is to find the city which is most dangerous among these four cities for having attack from all of the gangs X, Y and Z.

1) The combined choice matrices of X,Y and Z in different forms are,

$$e_A \begin{pmatrix} & & e_{B\wedge C} & \\ (0,1) & (1,0) & (0,1) & (1,0) \\ (0,1) & (1,0) & (0,1) & (1,0) \\ (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (1,0) & (0,1) & (1,0) \end{pmatrix} \quad [\text{ Since } \; B \wedge C = \{e_2, e_4\}, A = \{e_1, e_2, e_4\}]$$

$$e_B \begin{pmatrix} & & e_{C\wedge A} & \\ (0,1) & (1,0) & (0,1) & (1,0) \\ (0,1) & (1,0) & (0,1) & (1,0) \\ (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (1,0) & (0,1) & (1,0) \end{pmatrix} \quad [\text{ Since } \; C \wedge A = \{e_2, e_4\}, B = \{e_1, e_2, e_4\}]$$

$$\begin{array}{cc} & e_{A\wedge B} \\ e_C & \begin{pmatrix} (0,1) & (0,1) & (0,1) & (0,1) \\ (1,0) & (1,0) & (0,1) & (1,0) \\ (1,0) & (1,0) & (0,1) & (1,0) \\ (1,0) & (1,0) & (0,1) & (1,0) \end{pmatrix} \end{array} \quad [\text{ Since } A\wedge B=\{e_1,e_2,e_4\}, C=\{e_2,e_3,e_4\}]$$

2) Corresponding product intuitionistic fuzzy soft matrices are,

$$\begin{array}{cc} & e_A \\ U_A & \begin{pmatrix} (0.8,0.1) & (0.7,0.1) & (0,1) & (0.9,0.1) \\ (0.4,0.5) & (0.3,0.7) & (0,1) & (0.4,0.5) \\ (0.6,0.3) & (0.4,0.3) & (0,1) & (0.6,0.2) \\ (0.7,0.1) & (0.5,0.2) & (0,1) & (0.7,0.2) \end{pmatrix} \end{array} \otimes \begin{array}{cc} & e_{B\wedge C} \\ e_A & \begin{pmatrix} (0,1) & (1,0) & (0,1) & (1,0) \\ (0,1) & (1,0) & (0,1) & (1,0) \\ (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (1,0) & (0,1) & (1,0) \end{pmatrix} \end{array}$$

$$= \begin{pmatrix} (0,1) & (0.9,0.1) & (0,1) & (0.9,0.1) \\ (0,1) & (0.4,0.5) & (0,1) & (0.4,0.5) \\ (0,1) & (0.6,0.2) & (0,1) & (0.6,0.2) \\ (0,1) & (0.7,0.1) & (0,1) & (0.7,0.1) \end{pmatrix},$$

$$\begin{array}{cc} & e_B \\ U_B & \begin{pmatrix} (0.7,0.2) & (0.6,0.1) & (0,1) & (0.8,0.1) \\ (0.3,0.5) & (0.3,0.7) & (0,1) & (0.4,0.5) \\ (0.5,0.3) & (0.5,0.3) & (0,1) & (0.6,0.4) \\ (0.6,0.2) & (0.4,0.2) & (0,1) & (0.6,0.2) \end{pmatrix} \end{array} \otimes \begin{array}{cc} & e_{C\wedge A} \\ e_B & \begin{pmatrix} (0,1) & (1,0) & (0,1) & (1,0) \\ (0,1) & (1,0) & (0,1) & (1,0) \\ (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (1,0) & (0,1) & (1,0) \end{pmatrix} \end{array}$$

$$= \begin{pmatrix} (0,1) & (0.8,0.1) & (0,1) & (0.8,0.1) \\ (0,1) & (0.4,0.5) & (0,1) & (0.4,0.5) \\ (0,1) & (0.6,0.3) & (0,1) & (0.6,0.3) \\ (0,1) & (0.6,0.2) & (0,1) & (0.6,0.2) \end{pmatrix},$$

$$U_C \begin{array}{c} e_C \\ \begin{pmatrix} (0,1) & (0.8,0.1) & (0.2,0.7) & (0.9,0.1) \\ (0,1) & (0.3,0.7) & (0.9,0.1) & (0.4,0.5) \\ (0,1) & (0.5,0.2) & (0.6,0.3) & (0.7,0.1) \\ (0,1) & (0.6,0.2) & (0.4,0.5) & (0.6,0.2) \end{pmatrix} \end{array} \otimes e_C \begin{array}{c} e_{A\wedge B} \\ \begin{pmatrix} (0,1) & (0,1) & (0,1) & (0,1) \\ (1,0) & (1,0) & (0,1) & (1,0) \\ (1,0) & (1,0) & (0,1) & (1,0) \\ (1,0) & (1,0) & (0,1) & (1,0) \end{pmatrix} \end{array}$$

$$= \begin{pmatrix} (0.9,0.1) & (0.9,0.1) & (0,1) & (0.9,0.1) \\ (0.9,0.1) & (0.9,0.1) & (0,1) & (0.9,0.1) \\ (0.7,0.1) & (0.7,0.1) & (0,1) & (0.7,0.1) \\ (0.6,0.2) & (0.6,0.2) & (0,1) & (0.6,0.2) \end{pmatrix}$$

3) The sum of these product intuitionistic fuzzy soft matrices is,

$$\begin{pmatrix} (0,1) & (0.9,0.1) & (0,1) & (0.9,0.1) \\ (0,1) & (0.4,0.5) & (0,1) & (0.4,0.5) \\ (0,1) & (0.6,0.2) & (0,1) & (0.6,0.2) \\ (0,1) & (0.7,0.1) & (0,1) & (0.7,0.1) \end{pmatrix} \oplus \begin{pmatrix} (0,1) & (0.8,0.1) & (0,1) & (0.8,0.1) \\ (0,1) & (0.4,0.5) & (0,1) & (0.4,0.5) \\ (0,1) & (0.6,0.3) & (0,1) & (0.6,0.3) \\ (0,1) & (0.6,0.2) & (0,1) & (0.6,0.2) \end{pmatrix}$$

$$\oplus \begin{pmatrix} (0.9,0.1) & (0.9,0.1) & (0,1) & (0.9,0.1) \\ (0.9,0.1) & (0.9,0.1) & (0,1) & (0.9,0.1) \\ (0.7,0.1) & (0.7,0.1) & (0,1) & (0.7,0.1) \\ (0.6,0.2) & (0.6,0.2) & (0,1) & (0.6,0.2) \end{pmatrix} = \begin{pmatrix} (0.9,0.1) & (0.9,0.1) & (0,1) & (0.9,0.1) \\ (0.9,0.1) & (0.9,0.1) & (0,1) & (0.9,0.1) \\ (0.7,0.1) & (0.7,0.1) & (0,1) & (0.7,0.1) \\ (0.6,0.2) & (0.7,0.1) & (0,1) & (0.7,0.1) \end{pmatrix}$$

$$= R_f$$

4) Now the weights of the cities are respectively,

- $W(C_1) = 0.9 + 0.9 + 0 + 0.9 = 2.7$
- $W(C_2) = 0.9 + 0.9 + 0 + 0.9 = 2.7$
- $W(C_3) = 0.7 + 0.7 + 0 + 0.7 = 2.1$
- $W(C_4) = 0.6 + 0.7 + 0 + 0.7 = 2.0$

5) Now the cities C_1, C_2 associated with the first and second rows of the resultant intuitionistic fuzzy soft matrix(R_f) have the same highest weight $(W(C_1) = W(C_2) = 2.7)$, therefore

we have to consider the Θ-values of C_1 and C_2.

6) The Θ-values of C_1 and C_2 are respectively,

- $\Theta(C_1) = 0.1+0.1+1+0.1 = 1.3$
- $\Theta(C_2) = 0.1+0.1+1+0.1 = 1.3$

Since both the cities C_1 and C_2 have the same Θ-value(1.3), both of them have the same danger of terrorist attack. Hence the cities C_1 and C_2 are most dangerous among the four cities for having attack from all of the gangs X, Y and Z.

6 Conclusion:

In this paper first we have proposed the concept of intuitionistic fuzzy soft matrix and defined different types of matrices in intuitionistic fuzzy soft set theory. Then we have introduced here some new operations on these matrices and discussed here all these definitions and operations by appropriate examples. At last a new efficient solution procedure has been developed to solve intuitionistic fuzzy soft set based real life decision making problems which may contain more than one decision maker and to realize this procedure we also apply it to a more relevant subject of todays world as in Predict Terrorist Attack.

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